A perspective on empirical studies of contagion and interdependence*

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Abstract

This paper presents a factor-model approach to empirical test of contagion in financial markets. We show that leading tests in the literature are conditional on a specific yet arbitrary assumption about the ratio between the variance of the country-specific shock and the variance of the global factors weighted by factor loadings. We provide evidence on this ratio suggesting that, despite the claim by some authors, the issue of discriminating contagion and interdependence on empirical grounds is far from being settled.

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1 Introduction

In the past few years, currency and financial crises originating in one country or group of countries have often spread internationally. In periods of instability, asset price movements and comovements across markets and across borders have increased visibly relative to more tranquil periods. The size of these comovements during crises have led many economists to raise the question as of whether 'tranquil periods' and 'crises' are to be interpreted as different regimes in the international transmission of financial shocks; that is, as of whether there are discontinuities in the international transmission mechanism.

The possibility of such discontinuities are a concern for both investors and policy makers. If correlation across assets is abnormally high during financial crises, diversification of international portfolios may fail to deliver exactly when its benefits are needed the most. By the same token, excessive comovements of asset prices may spread a country-specific shock to other economies, even when these have better domestic fundamentals.

The headline of this theoretical and policy debate is usually referred to as 'contagion'. Contagion – as opposed to 'interdependence' – conveys the idea that international transmission mechanism is discontinuous, as a result of financial panics, herding, or switches of expectations across instantaneous equilibria. Although there is a considerable amount of ambiguity on what contagion exactly is and how we should measure it, several authors have proposed empirical tests in an attempt to address the issue of contagion versus interdependence on empirical grounds.

The idea underlying these studies is to compare cross-market correlation in tranquil and crisis periods and define *contagion as structural breaks in the parameters of the underlying data generating process*. For a given mechanism of international transmission, changes in the volatility of asset prices in one market can be expected to lead to changes in volatility in other markets. During a period of financial turmoil, therefore, some comovement across markets is an implication of interdependence. Contagion will occur when the observed pattern of comovement in asset prices is too strong, relative to what can be predicted when holding constant the mechanism of international transmission. Key to these studies is the specification of an appropriate theoretical measure of interdependence, suitable to capture the international effects of an increase in the volatility of asset prices for a given transmission mechanism.

This paper proposes a factor-model approach to the empirical analysis of contagion. Reviewing the literature within our general analytical framework, we show that many leading contributions derive measures of interdependence by making a specific yet arbitrary identification assumption about a key parameter. This is the ratio between the variance of the country-specific shock and the variance of the global factor weighted by its factor loading; we refer to it as the variance ratio, denoted by λ. Tests that tend to accept the null hypothesis of interdependence implicitly select a low value of λ, while tests that tend to reject the null of no contagion select a high value of λ. Preliminary estimates of λ shows that, in a number of cases, the null hypothesis of interdependence is erroneously accepted by existing tests, while it should be rejected in favor of contagion.

The paper is organized as follows. Section 2 presents a few stylized facts about stock market returns in the nineties, comparing their behavior during crisis and tranquil periods. A notable point here is that financial crises are characterized by an increase in the variance and covariance of returns across markets, but not necessarily by an increase in correlation. Section 3 introduces a factor model and derives a general empirical test. Section 4 discusses the existing literature in light of our model. Section 5 carries on a conditional test and presents some empirical evidence about financial contagion during the Hong Kong stock market crisis in October 1997.

2 Stylized facts

We start our analysis by presenting at a set of stylized facts regarding the transmission of shocks across stock markets.[1] Our data set includes 18 countries: the G7 countries, Argentina, Brazil, Mexico, Russia, Hong Kong, Indonesia, South Korea, Malaysia, Philippines, Singapore and Thailand. We use daily and weekly data, from January 1990 to March 2000; the source is *Thomson Financial Datastream.*

For each stock market in our sample, we examine levels and volatility of returns, calculated in local currency, as well as covariance and correlation patterns with other markets. We also calculate an indicator of coincidence in peaks during periods of turbulence. We allow for four periods of crisis in international financial markets: from September 1992 to August 1993 (hereafter *ERM crisis*), from October 1994 to June 1995 (hereafter *Mexican crisis*), from July 1997 to January 1998 (hereafter *Asian crisis*), and from May 1998 to March 1999 (hereafter *Russian/Brazilian crisis*). The emphasis of the study is however on stock markets of emerging economies during the second half of the 1990s.

2.1 Four empirical regularities

We single out four 'stylized facts' characterizing periods of international financial turmoil in our sample. The first three are: the concentration of sharp downward adjustments; the sharp increase in average volatility; and the sharp increase in the cross market *covariance* of assets' returns. Last, we show that the evidence about the direction of the change in cross market *correlation* of asset returns is inconclusive.

[1]In a companion paper, we present empirical evidence for nominal exchange rates against the U.S. dollar, overnight interest rates, and sovereign spreads of U.S. dollar denominated bonds with corresponding U.S. assets (see Corsetti et al., 2000).

2.1.1 Sharp falls in stock prices tend to concentrate in periods of international financial turmoil.

The performance of domestic stock markets is presented in the graphs 1a and 1b. A visual inspection of these graphs suggests different regional patterns. First, in the emerging economies of *Asia*, stock markets do not seem to be affected by the Mexican crisis. Stock indexes start to decline much before the eruption of the Asian crisis and stay on a descending path until the end of 1998. In *Latin America*, stock markets partially survive the Asian crisis, but are greatly affected by the Mexican crisis and by the Russian/Brazilian crisis. The impact of the last episode is especially strong and brings about a drop of over 50 per cent in stock indices. For most emerging market economies, stock prices at the end of March 2000 have not recovered relative to their historical level. In the *G7 countries*, the impact of the Mexican and the Asian crises is negligible, while the effect of the Russian/Brazilian crisis is much deeper. Yet stock markets recover quickly also after this crisis.

In order to disentangle the largest price movements, we calculate an indicator of price reversal as the ratio between the value of the stock market in period t and its maximum value up to period t $(x_t/max\{x_h\}_{h=0}^{t})$ — called $CMAX_t$. This indicator, commonly used by practitioners, is shown in Figures 1c and 1d. Note that regional differences are less pronounced in these graphs than in the previous two.

In the group of *emerging economies* as a whole, large downward movements in stock market indexes occur in correspondence with the Asian and the Russian/Brazilian crises. The price decline is considerable: it ranged from 50 per cent in Argentina to 90 per cent in Russia. Also in the *developed world*, with the sole exception of Japan, negative peaks somewhat concentrated during crisis periods. The Russian and Brazilian crises, in particular, are associated with a fall in asset prices between 20 and 30 per cent. Stock prices nonetheless recovered quickly, reaching new historical heights by the end of 1999.

2.1.2 Volatility of stock prices increases during crisis periods.

Volatility of stock market returns, shown in Figures 2a and 2b, increases almost everywhere in *Asia* in 1997-99 relative to 1990-96, with the sole exception of the Philippines.[2] In 1997 and 1998 volatility records two peaks, corresponding

[2] We compute 'instantaneous' volatility of returns at time t as an exponential-weighted moving average given by $\sigma_{i,t} = \sqrt{(1-\vartheta)\sum_{h=0}^{T-1}\vartheta^h(r_{i,t-h}-\overline{r}_i)^2}$ where ϑ is the decay factor (that we set equal to 0.96), T is the length of the moving window (that we pose equal to 3 months), $r_{i,h} = \log(x_{i,h}/x_{i,h-1})$ where $x_{i,h}$ is the value of the stock market index in country i at time h and $\overline{r}_i$ is the average of the variable $r_{i,h}$ in the period $[t-T+1,t]$. Analogously, instantaneous covariance is $\sigma_{ij,t} = (1-\vartheta)\sum_{t=0}^{T-1}\vartheta^h(r_{i,t-h}-\overline{r}_i)(r_{j,t-h}-\overline{r}_j)$ and the instantaneous correlation coefficient is $\rho_{ij,t} = \sigma_{ij,t}/(\sigma_{i,t}\sigma_{j,t})$. Volatility has also been estimated with a simple GARCH(1,1) model, which yields essentially the same results and, hence, it is not shown. Variables computed using daily, weekly and monthly returns give very similar results.

to the Asian and the Russian/Brazilian crises.[3] By contrast, Hong Kong is the sole country in the region that is significantly affected by the Mexican crisis. Overall, average volatility in 1997-99 is almost twice than in 1990-96. As regards *Latin American* countries, volatility of stock prices in the second half of the 1990s either decreases relative to previous record-high levels, as in the case of Argentina, or it remains constant, as in the cases of Brazil and Mexico. Volatility in these two countries is subject to large swings in correspondence with the crisis episodes — yet, it is around its sample average by the end of the decade. In *Russia*, volatility increases dramatically during the summer of 1998, reaching an instantaneous level of over 100 per cent.[4]

In *industrialized countries*, stock market volatility increases gradually from 1990 to 1999, with the exception of Japan, where it decreases, and France, where it remains steady. In most countries, volatility peaks in 1990-92, then decreases until 1997, when it peaks again, reaching historical heights in 1998.

To provide some evidence about the magnitude of changes in volatility between tranquil and crisis periods, we compare average squared daily returns.[5] Table 1 reports average volatility in each period of crisis relative to average volatility in tranquil periods, for emerging and developed countries.

For *emerging countries*, squared daily returns are below tranquil period averages during the ERM crisis, are about the same during the Mexican crisis (with the exception of Hong Kong, Brazil and Mexico), and reach enormous values during the last two crises. Similarly, squared daily returns *in developed countries* are in many cases below those of tranquil periods during the ERM and Mexican crisis, but are exceptionally large during the Asian and the Russian/Brazilian crises. On average, volatility during the last two episodes of crisis is more than sixty percent larger than during tranquil periods.[6]

2.1.3 Covariance between stock market returns increases during crisis periods.

Covariance of weekly returns, presented in Figure 3a, confirms that *Asian countries* are relatively unaffected by the Mexican crisis. Although covariance is never nil during this crisis (as is in most tranquil periods), its level is often lower than the peaks recorded before and after the crisis. Instead, the impact

[3] Only in Singapore and Thailand, volatility reached higher levels in 1990 than in 1997-99.

[4] Volatility of sovereign spreads followed a similar pattern during the period. It strongly increased in 1997 and in 1998, then gradually decreased in 1999 (see Corsetti et al., 2000).

[5] Averages are computed by the regression of $r_t^2 = \beta_0 D_{tranquil} + \beta_1 D_{ERM} + \beta_2 D_{Mexico} + \beta_3 D_{Asia} + \beta_4 X_{Russia/Brazil}$ where $D_{tranquil}$, D_{ERM}, D_{Mexico}, D_{Asia}, $D_{Russia/Brazil}$ are dummy variables that assume, respectively, unit value for the tranquil period, the ERM crisis, the Mexican crisis, the Asian crisis, the Russian and Brazilian crises. OLS estimates of the $\beta's$ represent averages in the corrispoding period. Table 1 reports estimates of $100 \cdot \left(\sqrt{\beta_i/\beta_{no-crisis}} - 1\right)$.

[6] Argentina ratios behave somewhat differently (especially in the last two crisis episodes) because of the dramatic increase in volatility that the stock market recorded at the beginning of the 1990s. Volatility was very large at the beginning of the 1990s also in other countries (notably, Japan, Indonesia and Thailand) due to the Gulf War and the recession in the United States.

of the Asian and the Russian/Brazilian crises on cross-country comovements of stock returns is striking. Covariance between weekly returns of Indonesia, Korea, Malaysia, the Philippines and Thailand obtains record-high during the Asian crisis, diminishes somewhat shortly after, and reaches new peaks in 1998-99. It comes back to normal levels only by the end of 1999. Covariances between each of this five countries with Hong Kong, Japan, Singapore and the U.S. follow a very similar pattern (Figures 3b and 3c).

In *Latin America*, covariances between returns of Argentina, Brazil and Mexico sharply increase sharply during the three episodes of crisis in the second half of the 1990s. Comovements of returns in Latin American countries with the *United States* are not significantly different from tranquil periods during the Mexican crisis, but are extremely high during the Asian and the Russian/Brazilian crises. Finally, covariances of Latin American countries and the United States with *Russia* (for which data is available only from January 1996) recorded sizable increments during the Asian and the Russian/Brazilian crises.

2.1.4 Correlation between stock market returns is not necessarily larger during crisis periods than during tranquil periods.

Figures 4a and 4b show correlation coefficients of weekly returns for the stock markets in the sample. For *Asian countries*, a first notable piece of evidence is the gradual increase in correlation after the beginning of the Asian crisis, and especially after the Hong Kong crash in October 1997. However, one cannot identify an analogous pattern during other episodes of crisis. For instance, correlation across Asian stock markets during the Russian/Brazilian crisis, either remains stable or decreases. By the same token, there is no single correlation pattern during the ERM and Mexican crises.

A second notable piece of evidence is that, even during the Asian crisis, correlation remains below or at the same level recorded between 1995 and 1997. As a result, correlation is not significantly higher during crisis periods than during tranquil periods.

In *Latin America*, correlation between the stock markets of Argentina, Brazil and Mexico increases during the Mexican, Asian and Russian/Brazilian crises; during the same crisis episodes, correlation of Latin American countries with the United States increases as well. Correlation between the *Russian* and the U.S. stock markets has gradually increased in the last six years.

As for *industrial countries*, correlation of the U.S. stock returns *vis-à-vis* France, United Kingdom, Italy and Canada is rising from the low values recorded in 1993-95, reaching a peak in 1999. Somewhat surprisingly, the correlation of both the United States and the European countries with Germany decreases from 1990 until end 1998. No clear trend is observable in the correlation between the Japanese and the US stock prices.

2.2 A synthesis through a case-study

Figure 5 below presents a case-study that summarizes well the typical patterns emphasized in our analysis above. The figure shows the pattern of CMAX, volatility, covariance and correlation for Hong Kong and the Philippines. While the Mexican crisis has a negligible impact on the Philippines (as well as on most of the other Asian countries), stock prices in Hong Kong record a sharper decline and an abrupt rise in volatility. As a result, cross-market linkages between the two countries weaken and both covariance and correlation decrease.

Yet, the increase in volatility and covariance during the Asian, and the Russian/Brazilian crises is quite striking. In particular, covariance between the two markets rises from nil to its decade-record high, with a sharp step up around October 1997, when the Hong Kong stock market plummet. Covariance remains on high levels until the first quarter 1998, then decreases somewhat, before rising again in correspondence with the Russian turmoil. Correlation increases steadily during the Asian crisis although it does not appear significantly larger than the levels reached in 1996; it decreases somewhat between May and September 1998 and is fairly stable thereafter.

In light of the stylized facts discussed above, what strikes market participants as evidence of contagion is the magnitude of asset price movements occurring more or less simultaneously in different regions of the world, as measured by the dramatic increase in covariance and volatility. Correlation seldom rises above the level recorded in tranquil periods, even during 'extreme' episodes of international transmission of shocks.

3 The model

3.1 A factor-model approach to the analysis of contagion

This section lays out a simple factor model to approach the issue of testing for structural breaks in the international transmission mechanism. For the purpose of comparison with the current literature, in what follows we focus on correlation analysis, casting our argument in the framework of a single factor model. A meaningful generalization of our argument to multi-factor models is best accomplished without using correlation-based tests – a task that is left to future contributions.

Assume that the rates of return of the stock markets in country i and country j are generated by the process

$$\begin{aligned} r_i &= \alpha_i + \gamma_i \cdot f + \varepsilon_i \\ r_j &= \alpha_j + \gamma_j \cdot f + \varepsilon_j \end{aligned} \tag{1}$$

where α_i and α_j are constant numbers, γ_i and γ_j are market-specific factor loadings, f is a global factor, ε_i and ε_j denote idiosyncratic risks, and where f, ε_i and ε_j are mutually independent random variables with finite and strictly positive variance.

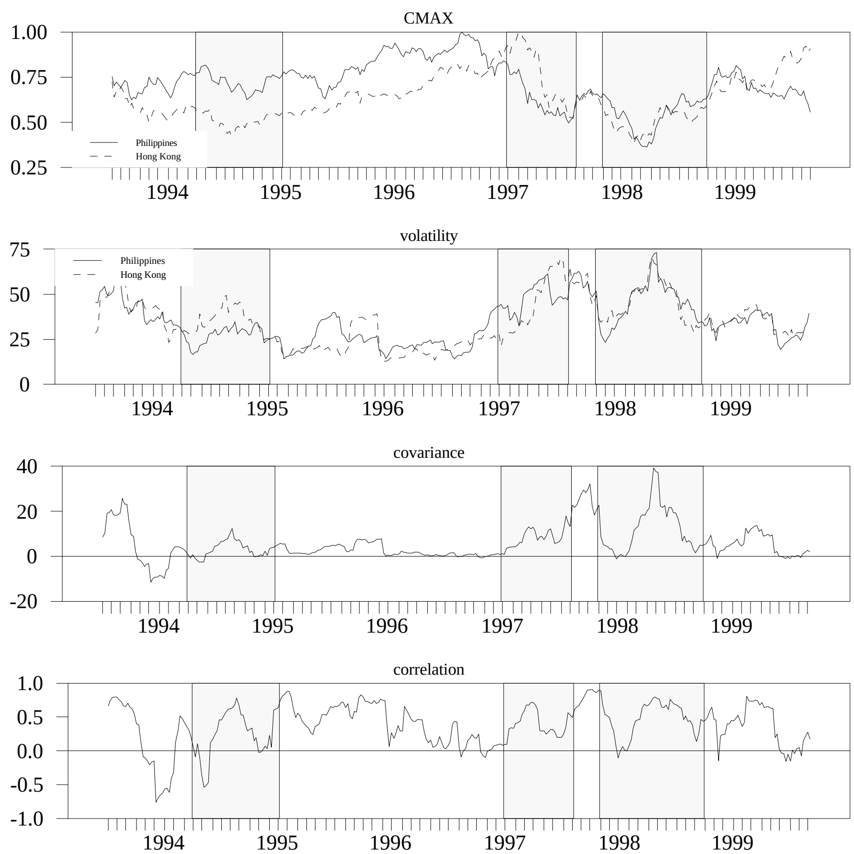

Figure 5: CMAX, instantaneous volatility, covariance and correlation between stock market returns of Hong Kong and the Philippines. Shaded areas correspond to the periods of crisis de…ned in the text.

For simplicity, let both γ_i and γ_j be strictly positive. From the process above, the correlation coefficient between r_i and r_j can be written as:[7]

$$\begin{aligned}
\rho &\equiv Corr(r_i, r_j) = \frac{Cov(r_i, r_j)}{\sqrt{Var(r_i) \cdot Var(r_j)}} \\
&= \frac{1}{\left[1 + \frac{Var(\varepsilon_i)}{\gamma_i^2 Var(f)}\right]^{1/2} \cdot \left[1 + \frac{Var(\varepsilon_j)}{\gamma_j^2 Var(f)}\right]^{1/2}}
\end{aligned}$$

Given the factor loadings γ_i and γ_j, a rise in correlation must correspond to shocks increasing the variance of the global factor f relative to the variance of the idiosyncratic noise ε_i and/or ε_j. Given the variances of the global factor and the idiosyncratic components, however, a rise in correlation could also correspond to an increase in the magnitude of the factor loadings γ_i and γ_j, or to an increase in the correlation between the idiosyncratic risks.

This distinction is at the root of recent empirical studies contrasting contagion to interdependence. Consider a financial crisis in country j. The increase in the variance of the stock market return in such a country may be due to an increase in the variance of either the global factor f, or the country specific component ε_j, or both. It is apparent that, if the change in the variance of the global factor f is large enough relative to the change in the variance of the country specific component ε_j, cross-market correlation must increase during a crisis in county j. This change in correlation is *interdependence*, in the sense that, *conditional on the occurrence of a financial crisis in country j, it is consistent with the data generating process* (1). *Contagion*, as opposed to interdependence, occurs if the increase in correlation turns out to be 'too strong' relative to what is implied by the process (1); i.e. it is too strong to be explained by the behavior of the global factor and the country specific component. In other words, contagion occurs when, conditional on a crisis, correlations are stronger because of some structural change in the international economy — affecting the link across markets.

In a related definition, contagion occurs when a country-specific shock becomes 'regional' or 'global'. This means that there is some factor η for which factor loadings are zero in all countries but one during tranquil periods, and become positive during crisis periods. An illustration of this concept of contagion is provided by the following two-factor model:

$$\begin{aligned}
r_i &= \alpha_i + \gamma_i \cdot f + \beta_i \cdot \eta + \varepsilon_i \\
r_j &= \alpha_j + \gamma_j \cdot f + (\eta + \eta_j)
\end{aligned}$$

where β_j has been normalized to 1. If interdependence, $\beta_i = 0$, so that the process is equivalent to the data generating process (1) by setting $\varepsilon_j = \eta + \eta_j$. Contagion occurs when the country specific shock η becomes a global factor, *i.e.* when $\beta_i \neq 0$. As shown below, our measure of interdependence is derived

[7] We denote with Var the variance operator, Cov the covariance operator and $Corr$ the linear correlation operator.

under the null hypothesis $\beta_i = 0$. Thus, it will be unaffected by a change in the specification of the process for the rates of return, which uses the above expressions instead of the process (1).

These definitions provide a general framework for the empirical test discussed below.

3.2 Conditional correlation analysis

How can one derive a theoretical measure of correlation suitable to discriminate between contagion and interdependence according to the model presented above? Suppose that we can identify the 'origin' of an international financial crisis in some country j (e.g., Mexico at the end of 1994, Thailand in July 1997, Hong Kong in October 1997). Let δ denote the proportional change in the variance of the stock market return r_j relative to the pre-crisis period. The, we can write

$$Var(r_j \mid C) = (1+\delta)Var(r_j)$$

where C denotes the event 'crisis in country j'. Note that the observed change in the variance of r_j does not necessarily coincide with an increase in the variance of the global factor, as the variance of the country-specific component may also change during the crisis.

In order to test whether changes in the correlation between r_i and r_j during a crisis in j are consistent with the data generating process (1), we must specify a measure of interdependence under the assumption that γ_i, γ_j, $Var(\varepsilon_i)$ and $Cov(\varepsilon_i, \varepsilon_j)$ *do not* vary with the crisis in country j. In the Appendix we show that, under such an assumption, the correlation coefficient between r_i and r_j can be written as the following function ϕ:

$$\phi(\lambda_j, \lambda_j^C, \delta, \rho) \equiv \rho \left[\left(\frac{1+\lambda_j}{1+\lambda_j^C} \right)^2 \frac{1+\delta}{1+\rho^2 \left[(1+\delta) \frac{1+\lambda_j}{1+\lambda_j^C} - 1 \right] (1+\lambda_j)} \right]^{1/2} \quad (2)$$

where λ_j (λ_j^C) denotes the ratio between the variance of the idiosyncratic shock ε_j and the variance of the global factor f, scaled by the factor loading γ_j, during the tranquil (crisis) period:

$$\lambda_j = \frac{Var(\varepsilon_j)}{\gamma_j^2 \cdot Var(f)} \text{ and } \lambda_j^C = \frac{Var(\varepsilon_j \mid C)}{\gamma_j^2 \cdot Var(f \mid C)} \ .$$

In what follows, we will refer to ϕ as a theoretical measure of interdependence. The correlation coefficient between r_i and r_j observed during the crisis, denoted by ρ^C, and the theoretical measure of interdependence ϕ are the main elements of our test.

The coefficient ϕ is derived under the null hypothesis of interdependence: if γ_i, γ_j $Var(\varepsilon_i)$ and $Cov(\varepsilon_i, \varepsilon_j)$ do not change during the crisis, ρ^C and ϕ will coincide. Conversely, if there is contagion in the form of an increase in the magnitude of factor loadings or a positive correlation between idiosyncratic risks (e.g., because some country-specific factor becomes global during the crisis in country j), ρ^C will be larger than ϕ. Then, under the identifying assumption that contagion from international crises does not alter the variance of idiosyncratic shocks in countries other than j (i.e. $Var(\varepsilon_i)$ is constant), a statistical analysis on contagion vs. interdependence can be performed by testing whether ρ^C is significantly higher than ϕ.

We should stress a notable feature of this approach to testing. During an international crisis originating in one country, shocks to the global factor tend to induce large comovements of prices. Yet, the country where the crisis originates may also be subject to large shocks that are and remain country-specific. Overall cross-market correlation may fall. The fact that during a crisis correlation falls (as it often does in the data, see Section 2) is by no means evidence against contagion. In other words, testing for contagion needs not be conditional on observing a hike in correlation. In line with this remark, our test procedure is symmetrical; namely, it can also be applied to structural breaks and contagion consisting in looser interdependence (e.g. falling factor loadings). There is no reason why the concept of contagion should be confined to the hypothesis of stronger than normal ties.

4 A review of the literature

This section analyzes recent empirical contributions on contagion, identifying a set of tests that can be interpreted as special cases of our framework. To introduce our discussion, it is useful to simplify our test statistic ϕ by assuming that the variance ratio defined in the previous section does not vary across periods, $\lambda_j^C = \lambda_j$. Assuming a constant ratio means that the variance of the global factor and the variance of the country-specific risk increase by the same proportion during the crisis in j:

$$\frac{Var(r_j \mid C)}{Var(r_j)} = \frac{Var(f \mid C)}{Var(f)} = \frac{Var(\varepsilon_j \mid C)}{Var(\varepsilon_j)} = 1 + \delta$$

Then, the coefficient of interdependence ϕ simplifies to:

$$\phi(\lambda_j, \delta, \rho) = \rho \left[\frac{1+\delta}{1+\delta\rho^2(1+\lambda_j)} \right]^{1/2} \tag{3}$$

Other things equal, a larger variance-ratio λ_j reduces the effect of an increase in the variance of r_j on the coefficient of interdependence. This is because a larger fraction of this variance is due to the country-specific component, hence weakening cross-market linkages.

To clarify this point, we present some first evidence on one of the case-studies that will be analyzed in detail in a later section of this paper. This is the spread of financial instability in the stock market from Hong Kong to the Philippines, on

October 1997. Figure 6 below shows the 'instantaneous' correlation coefficient between stock market returns in Hong Kong and the Philippines, both measured in US dollars, during 1997. The daily correlation provides a proxy for ρ (during tranquil periods) and ρ^C (during crises). Note that, before October 20, which is the starting day of the crisis, we only report the instantaneous correlation, ρ_t; from October 20 on, we report both the instantaneous correlation, ρ_t^C, and a set of coefficients of instantaneous correlation under the null hypothesis of interdependence, calculated assuming different values of λ_j. For the purpose of the graph, we find it useful to calculate and plot an inverse transformation of ϕ, instead of ϕ itself. This transformation, denoted by $\phi_t^{'}(\lambda_j)$ is given below

$$\phi_t^{'}(\lambda_j) = \frac{\rho_t^C}{\sqrt{1 + \hat{\delta} - \hat{\delta}\left(\rho_t^C\right)^2 - \hat{\delta}\lambda_j\left(\rho_t^C\right)^2}}$$

where $\hat{\delta}$ is estimated from the sample data.[8]

According to the logic of our test, this coefficient of correlation is adjusted so as to allow for the fact that changes in the volatility of stock prices in Hong Kong will *per se* affect cross-border comovements during the Hong Kong crisis. Thus, the observed ρ_t^C is adjusted on the basis of the estimated increase in the variance of r_j, that is $\hat{\delta}$. Given $\hat{\delta}$, a smaller λ_j (shifting weight towards an increase in the variance of the global factor) entails a smaller adjusted coefficient.

A visual inspection of figure 6 suggests that the unadjusted correlation coefficient ρ_t^C increased significantly during the Hong Kong crisis in October 1997 relative to the previous months. Is this evidence of contagion? In light of what discussed in the previous section, we can test of contagion vs. interdependence by comparing $\phi_t^{'}(\lambda_j)$ and ρ_t. Specifically, the null hypothesis of interdependence is accepted when $\phi_t^{'}(\lambda_j)$ is not significantly larger than ρ_t. Figure 6 plots different estimates of $\phi_t^{'}(\lambda_j)$ conditional on values of λ_j between 0 and 5. The graph shows that the adjusted coefficient $\phi_t^{'}(\lambda_j)$ is close to ρ_t for low values of the variance ratio, while it gets significantly larger for values of λ_j around 5. The graph suggests that the hypothesis of interdependence could be accepted conditional on some λ_j smaller than 5.

The literature provides a few conditional correlation tests of contagion — but in most cases the maintained assumption on the $\lambda's$ is only implicit. In the following section, we will review these tests, nesting them in our framework.

4.1 Tests based on sample correlation coefficient or $\lambda = 1/\rho^2 - 1$

Early contributions on contagion, such as King and Wadhwani (1990), acknowledge the problem of controlling for the relationship between volatility of return

[8] The coefficient ϕ' is obtained by substituting ϕ with ρ^C in equation (2), and then solving the resulting expression for ρ.

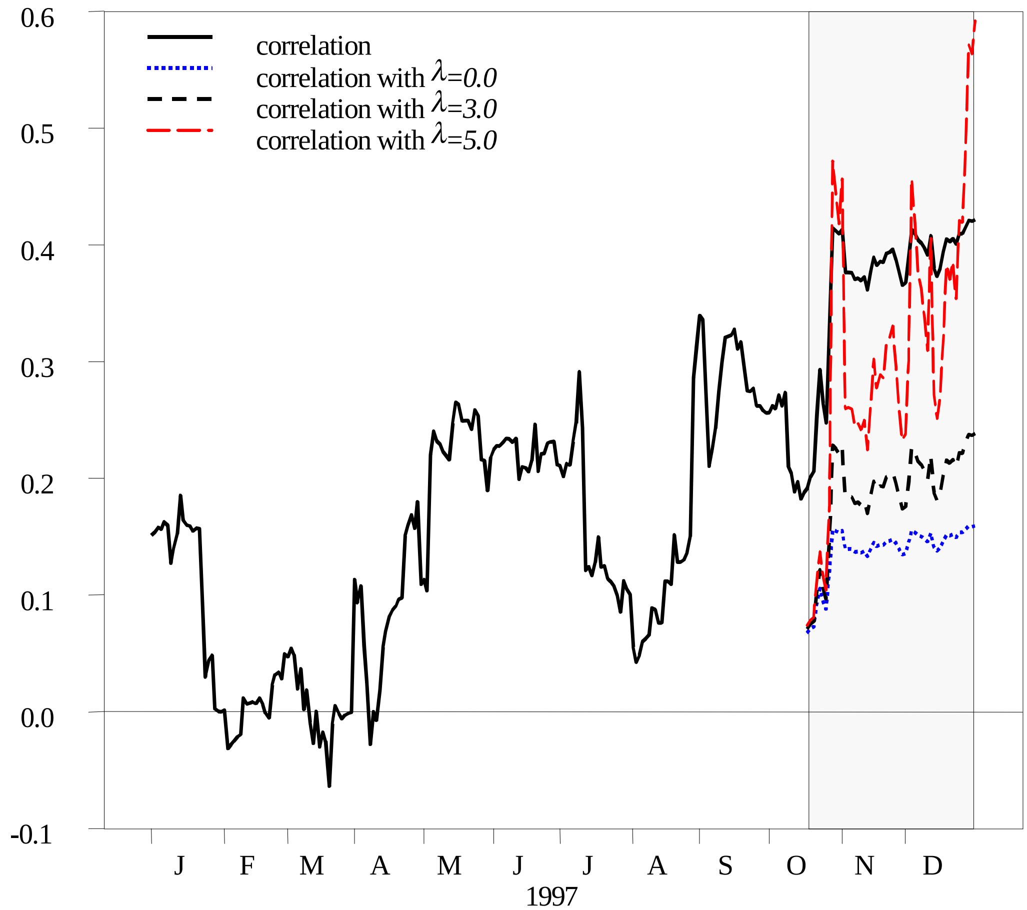

Figure 6: Instantaneous correlation and corrected correlation for di¤erent values of ¸ j between stock market returns in US dollars of Hong Kong and the Philippines

and correlation, but implement no correction of their empirical tests.[9] It is instructive to use our model in order to highlight the conditions under which a simple test on correlation is consistent with our measure of interdependence. Looking at equation (3), note that ϕ is identically equal to ρ only when

$$\lambda_j^C = \lambda_j = 1/\rho^2 - 1 \ . \tag{4}$$

For this particular value of the variance ratio,[10] interdependence implies that the correlation coefficient should not respond to a crisis in country j. Thus, we can perform a test of contagion just verifying whether the simple correlation coefficient has changed significantly during a crisis.

Interestingly, the implicit assumption in condition (4) is one of a negative relationship between the correlation coefficient during tranquil period ρ and the variance ratio λ_j: the higher the correlation between r_i and r_j, the higher the importance of the global factor and, in turn, the lower λ_j. This is not an unreasonable assumption in general. However, unless λ_j happens to be exactly equal (or close) to the inverse of the squared correlation coefficient minus one, tests of contagion based on comparing simple correlation will be biased — it could be interesting to explore the loss of accuracy of the test in the region around that value of the variance ratio.

4.2 Tests based on adjusted correlation coefficient with $\lambda = 0$

Consider the approach championed by Forbes and Rigobon (1999a,b). The key to these contributions is the (implicit) assumption that the rate of return of the stock market in country j coincides with a global factor. In terms of our factor model, this is equivalent to assuming that the data generating process of the rates of return is:

$$\begin{aligned} r_i &= \alpha_i + \gamma_i \cdot f + \varepsilon_i \\ r_j &= \alpha_j + \gamma_j \cdot f \end{aligned} \tag{5}$$

so that

$$r_i = \left(\alpha_i - \frac{\alpha_j}{\gamma_j}\right) + \frac{\gamma_i}{\gamma_j} \cdot r_j + \varepsilon_i$$

[9] King and Wadhwani (1990) are aware of the relationship between volatility and correlation as they write: "we might expect that the contagion coefficients would be an increasing function of volatility" (pp. 20). However, in calculating correlation between markets, they do not correct for the increase in volatility.

[10] A similar but more cumbersome expression could be derived for the general case in which $\lambda_j^C \neq \lambda_j$.

corresponding to the linear equation at the root of Forbes and Rigobon' estimates:[11]

$$r_i = \beta_0 + \beta_1 \cdot r_j + \varepsilon_i \tag{6}$$

Thus, there is no country-specific shock affecting r_j. In terms of our framework, $Var(\varepsilon_j) = 0$ implies $\lambda_j^C = \lambda_j = 0$.

A statistical framework closely related Forbes and Rigobon's test is presented by Boyer et al. (1999) and Loretan and English (2000), who assume that (r_i, r_j) is a normal bivariate random variable. The equivalence between the two approaches can be easily understood by referring to the following property: if (r_i, r_j) is a normal bivariate random variable, one can write

$$\begin{aligned} r_i &= \alpha_i + \gamma_i \cdot r_j + \upsilon_i \\ r_j &= \alpha_j + \gamma_j \cdot \upsilon_j \end{aligned} \tag{7}$$

where υ_i and υ_j are orthogonal and normally distributed random variables. It is apparent that, as in Forbes and Rigobon, the country-specific shock in j is the global factor, up to an affine transformation.

The test statistics adopted by Boyer et al. (1999) and Loretan and English (2000), which follows from the model (7), is the same as the one adopted by Forbes and Rigobon (1999a,b):

$$\rho \left[\frac{1+\delta}{1+\delta\rho^2} \right]^{1/2} \tag{8}$$

This statics represents the correlation between two jointly normal random variables as a function of the increase in the variance of one of them, δ, and it is known in the literature as 'normal correlation theorem'.[12] Note that it coincides with our measure of interdependence (3) when there is no idiosyncratic shock in country j; that is, when $\lambda_j^C = \lambda_j = 0$. Thus, the measures of interdependence (2) and (3) could be interpreted as a generalization of the normal correlation theorem.

In these models, the test strategy consists in verifying whether the statistics (8) is significantly different from ρ^C. The drawback of tests using the statistics (8) is quite clear. In equation (6), r_i depends linearly on r_j, so that there is no component of the variance of r_j that is country-specific. The stock market return in country j is specified as a 'global' factor, or — perhaps more precisely — as a 'regional' factor. The test statistic (8) is therefore only applicable when every single shock in country j has global or regional repercussions. Do we

[11]Forbes and Rigobon (1999a) filter their data estimating a VAR model with domestic and international interests rates and lagged returns. Then, they analyze the correlation between the residuals of their estimates with the model (6). Note that in the theoretical part of the paper Forbes and Rigobon (1999a) actually write a symmetric model, where r_i and r_j are interdependent. However, the symmetric model is not estimated.

[12]The correlation coefficient introduced in the 'normal correlation theorem', as presented in Boyer et al. (1999) and Loretan and English (1999) has a slightly different form. However, after some simple algebra it can be rearranged into the expression (8).

really believe that the rate of return in Hong Kong or Thailand is a global or even a regional factor?

The specification of r_j as a global factor has important implications for the test. To the extent that the increase in the variance of the market in country j is due to idiosyncratic shocks in this country, the theoretical correlation coefficient (8) will be biased. Such bias will be larger, the larger the share of variance in r_j that can be attributed to country-specific shocks. As apparent from equation (6), specifying r_j as a global factor magnifies the theoretical correlation ϕ between the two markets, and increases the chances that its variance will explain the observed correlation during the crisis. Hence, the test will be biased towards the null hypothesis of interdependence. It may not come entirely as a surprise that *this kind of tests hardly find any evidence of contagion.*

5 Empirical evidence

In this section we present an application of our methodology to the international effects of the October 1997 stock market crisis in Hong Kong.[13] Using data from *Thomson Financial Datastream*, we analyze correlation between stock market returns of Hong Kong with ten emerging economies (Indonesia, Korea, Malaysia, the Philippines, Singapore, Thailand, Russia, Argentina, Brazil and Mexico) and the G7 countries. In our benchmark estimation we calculate two-day rolling averages of daily returns in US dollars and we define tranquil and turbulent periods as starting from January 1 1997 to October 17 1997 and from October 20 1997 to November 30 1997 respectively.

We utilize US dollar returns because they represent profits of investors with international portfolios. As stock markets in different countries are not simultaneously open, two-day rolling averages of returns have been preferred to simple returns. The definition of crisis period follows the crash recorded by the stock market index in Hong Kong, which lost 25 per cent of its value in just four days starting on October 20 1997. Hong Kong stock prices declined until the end of November, apparently influencing returns in several other markets.

In order to verify the impact of these somewhat arbitrary choices, we carry out robustness tests. Specifically, we re-run our tests using data in local currency, modifying the definitions of tranquil and crisis periods, replacing rolling averages of returns with simple daily returns, and filtering the data with interest rates in the U.S.. Our main results are robust across all these alternative estimations.

Although our test procedure is symmetrical, we adopt the common practice of testing for contagion as a phenomenon in which correlation is significantly higher during the crisis period. Hence, our test hypotheses are:

$$\begin{aligned} H_0 &: \rho^C \leq \phi \qquad \textit{interdependence} \\ H_1 &: \rho^C > \phi \qquad \textit{contagion}\ . \end{aligned}$$

[13] For a comparison, see Forbes and Rigobon (1999a).

Looking at the definition (2) of the coefficient of interdependence ϕ, note that ρ and δ, as well as the coefficient ρ^C, can be easily estimated from the data. The main challenge in carrying out this test is to find good estimates of λ_j and λ_j^C.

We will proceed as follows. First, we set up a conditional test fixing the value of variance ratios parametrically; namely, we calculate minimum thresholds for λ_j and λ_j^C at which the difference between ρ^C and ϕ becomes statistically significant. Second, we compare these threshold with empirical estimates of these variance ratios, obtained using different methods. Finally, we carry on some robustness tests.

5.1 Conditional test: identification of threshold values for λ and λ^C

In this section, we identify critical thresholds for λ_j and λ_j^C at which the null hypothesis is rejected at a given confidence level. To clarify the meaning of these thresholds, consider first the case in which $\lambda_j^C = \lambda_j$. By inspecting equation (3), we see that ϕ is monotonically decreasing in λ_j, for given ρ and δ. Suppose we find ρ^C significantly larger than ϕ for a given $\lambda_j = \lambda'$; it follows that ρ^C is significantly larger than ϕ also for any $\lambda_j = \lambda''$, with $\lambda'' > \lambda'$. Therefore, we can look for *the minimum value of* λ_j — denoted with $\bar{\lambda}$ — at which the hypothesis of interdependence would be rejected at some prespecified confidence level. Analogously, in the case $\lambda_j^C \neq \lambda_j$, equation (2) shows that ϕ is monotonically decreasing in λ_j^C. Hence, for any given λ_j we can look for *the minimum value of* λ_j^C, $\bar{\lambda}^C$, at which the hypothesis of interdependence would be rejected. In the first case, the result of the conditional test will be a *threshold* $\bar{\lambda}$; in the second case, the result will be a *threshold function*, that gives the threshold $\bar{\lambda}^C$ for any $\lambda_j \geq 0$.

Tests of equality between two correlation coefficients can be performed using the *Fisher z-transformation*

$$z(\widehat{\rho}) = \frac{1}{2} \ln \frac{1 + \widehat{\rho}}{1 - \widehat{\rho}}$$

where $\widehat{\rho}$ is the estimated correlation coefficient. Under the assumption that two samples are drawn from two independent bivariate normal distributions with the same correlation coefficient, Stuart and Ord (1991, 1994) show that the difference $z(\widehat{\rho}_1) - z(\widehat{\rho}_2)$, where $\widehat{\rho}_1$ and $\widehat{\rho}_2$ are estimated correlation coefficients, converges to the following distribution:

$$N\left(0, \frac{1}{n_1 - 3} + \frac{1}{n_2 - 3}\right)$$

with n_1 and n_2 denoting the size of the two samples.

We proceed as follows. We estimate the correlation during the tranquil period, $\widehat{\rho}$, and during the crisis period, $\widehat{\rho}^C$, as well as the increase in the variance in the Hong Kong stock market, $\widehat{\delta}$. By substituting $\widehat{\rho}$ and $\widehat{\delta}$ into (2), we obtain an estimation of our measure of interdependence as a function of λ_j and λ_j^C, that is $\widehat{\phi}(\lambda_j, \lambda_j^C)$. Given $z(\widehat{\rho}^C)$ and $z(\widehat{\phi}(\lambda_j, \lambda_j^C))$, we derive threshold values of λ_j and λ_j^C from:

$$z(\widehat{\rho}^C) - z(\widehat{\phi}(\bar{\lambda}, \bar{\lambda}^C)) = 1.645\sigma_z \tag{9}$$

where $\sigma_z = \frac{1}{n-3} + \frac{1}{n^C-3}$, with n and n^C denoting the sample size of the tranquil and the crisis period.

If our samples were independent, the confidence level associated to threshold values derived from equation (9) would be 5%. However, in our test, the assumption of independent sample is violated, because $\widehat{\delta}$ depends on both the tranquil and the crisis period samples. To assess the distortion in our testing procedure, we have performed Montecarlo simulation experiments. Specifically, we have run $1,000,000$ replications for different country pairs, varying the parameter values and sample size. For instance, setting $n = 208$, $n^C = 30$ and $\delta = 8.72$, as in our benchmark estimation, and $\rho = 0.219$, $\rho^C = 0,661$, which are the observed correlation coefficients between the markets of Hong Kong and the Philippines, the significance level of the test corresponding to (9) is 8.1 per cent instead of 5 per cent. In all our simulations, the significance level of the statistics (9) is included within the range between 7 and 9 per cent. We conclude that the size of the distortions is not too large, as the confidence level associated with (9) does not exceed 10%.

5.1.1 Constant variance ratio

Consider first the case $\lambda_j = \lambda_j^C$. The threshold level of the variance ratio, $\bar{\lambda}$, can be easily found by inverting equation (9). This yields

$$\bar{\lambda} = \left\{ \left[\widehat{\rho} \frac{\widehat{\omega}+1}{\widehat{\omega}-1} \right]^2 \left(1+\hat{\delta}\right) - 1 \right\} \frac{1}{\hat{\delta}\widehat{\rho}^2} - 1.$$

where $\widehat{\omega} = \exp\left[2\left(z(\widehat{\rho}^C) - 1.645\sigma_z\right)\right]$. Consistently with the logic of our test, for given the sample estimates of correlations, if one believes that the variance ratio in Hong Kong during 1997 were constant and lower than the value $\bar{\lambda}$ solving the above equation, one should also accept the null hypothesis of interdependence.

The first two columns of table 2 report the correlation between two-day rolling averages of stock market returns in US dollars of Hong Kong with each country in the sample during the tranquil, $\widehat{\rho}$, and the crisis period, $\widehat{\rho}^C$. The third column of table 2 reports the threshold level of the variance ratio, $\bar{\lambda}$. It is apparent that $\bar{\lambda}$ tends to be larger, the smaller the difference between $\widehat{\rho}^C$ and $\widehat{\rho}$; in other words, if the correlation between two stock markets does not increase sharply during the crisis period, the null of interdependence can be rejected only for very high values of the variance ratio. Note also that, when correlation decreases between the tranquil and the crisis period, the null of interdependence cannot be rejected at all ($\bar{\lambda} = +\infty$). When the correlation in the tranquil period is about zero, as in the case of Italy, the null of interdependence is rejected for any value of λ_j.

Based on this simple calculations, table 2 shows that the null hypothesis of interdependence will be rejected for 'low' values of λ_j in the case of Italy, France, Singapore, the UK, and the Philippines. For instance, if one believes that $\lambda_j = 3$ (a value that we will find in our estimates), our test would reject interdependence for all the countries listed above . For $\lambda_j = 7$ (that will be our highest estimated value), the test would reject also for Germany. Note that setting $\lambda_j = \lambda_j^C = 0$ (as done in some of the literature), the test would reject interdependence only in the case of Italy.

For the sake of comparison, table 2 also reports the results of the Fisher test, where the null hypothesis under investigation is $H_o : \rho^C \leq \rho$. We have shown that this test corresponds to our conditional correlation analysis if $\lambda_j = 1/\rho^2 - 1$. In the table, observe that whenever $1/\rho^2 - 1 > \bar{\lambda}$ the test rejects the null. More interestingly, the value of $1/\rho^2 - 1$ (that is, the variance ratio at which a simple test on correlation coefficient is consistent with our framework) are quite high. Only for two countries, Singapore and Indonesia, it is smaller then 10. Recall that, since there is at most one value of the variance ratio that is true for Hong Kong, the test will be correct for at most one of the country pairs.

5.1.2 Variable variance ratio

Allowing the variance ratio to vary between the tranquil and the crisis period, i.e. $\lambda_j^C \neq \lambda_j$, equation (9) can be rewritten as

$$\left[1 + \widehat{\rho}^2 \frac{\widehat{\delta}\,(1+\lambda_j) + \left(\lambda_j - \bar{\lambda}^C\right)}{\left(1+\bar{\lambda}^C\right)} (1+\lambda_j)\right] \left(\frac{1+\bar{\lambda}^C}{1+\lambda_j}\right)^2 - \\ - \left(\widehat{\rho}\frac{\widehat{\omega}+1}{\widehat{\omega}-1}\right)^2 \left(1+\widehat{\delta}\right) = 0.$$

which implicitly defines $\bar{\lambda}^C$ as a function of λ_j. Figure 7 below graphs this implicit function for the case of Hong Kong and the Philippines. For any pair (λ_j, λ_j^C) above the function, the test will reject the hypothesis of interdependence. For any pair (λ_j, λ_j^C) below the function, the test will accept the null. The pair at the crossing between the function and the 45° degree line from the origin, identifies the threshold $\bar{\lambda}$ reported in table 2.

5.2 Estimation of the variance ratio

What do we know about λ_j and λ_j^C? A very simple approach to estimating these variance ratios consists in using a composite 'global factor' derived as the daily average return in a cross section of stock markets. Using this approach, first, we calculate the global factor using both the sample of the G7 countries and

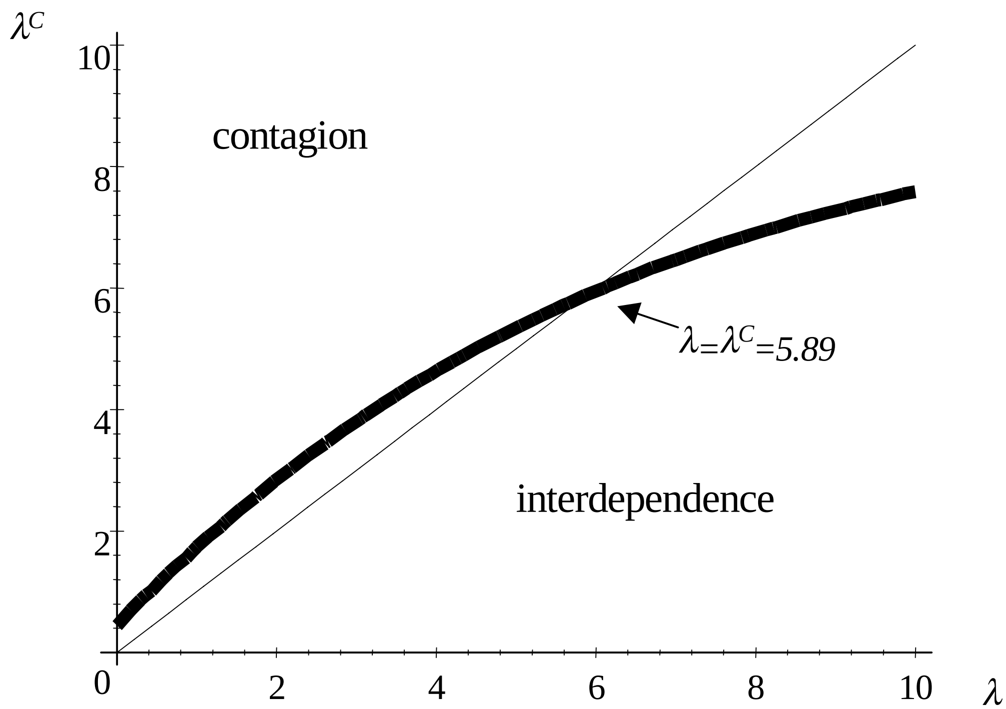

Figure 7: Hong Kong - Philippines threshold function

our full sample, but excluding Hong Kong. We also use the 'world stock market index' produced by *Thomson Financial Datastream*. Then, we compute the two-day rolling average of returns on the global factor, and we regress the two-day rolling average of Hong Kong's returns on it. The variance of the residuals from this regression gives an estimate of the variance of the country specific shock, which are used to estimate λ_j.

The results are shown in the first half of table 3. In our sample, the order of magnitude of the variance ratio for Hong Kong is between 2 and 4: i.e. in the Hong Kong stock market, the variance of country-specific shocks is between 2 and 4 times the variance of the global factor (multiplied by the factor loading γ_j). Most interestingly, these ratios do not vary substantially between the two sub-samples.

A second approach is based on principal component analysis. First, we calculate the principal components for our full sample of rolling averages of returns. We then regress the rolling average of returns in country j on the principal components, using the residual from this regression to estimate the variance of the country specific shocks. Results are shown in the second half of table 3.

When the tranquil and crisis periods are added together, our estimates of λ_j are very similar to what we have obtained by using the composite global factor. The variance ratio is between 4 and 7, depending on the number of components included in the regression. At the margin, higher estimated values of λ_j make a difference only for the case of Germany.

When we analyze the tranquil and the crisis period separately, the analysis of principal components provides quite a different picture. Including only one component in our regression, λ_j is as high as 17 in the tranquil period, but declines to 2.3 during the crisis. Under the null, this suggests a significant increase in the variance of the global factor, relative to the country-specific shock.

A notable conclusion from our preliminary (and admittedly rough) estimates, is that the variance ratio is well below what is needed to justify a test based on unadjusted correlation coefficients (see table 2). At the same time, however, the value of the $\lambda's$ is bounded away from zero. The strong results of interdependence reached by Boyer et al. (1999) and Forbes and Rigobon (2000) do not survive when the implicit bias in their test is removed.

5.3 Robustness tests

In order to test the robustness of our results, we re-run our analysis by changing the definition of our sample. We consider returns in local currency, as opposed to US dollars, we alter the definition of tranquil and crisis periods, and we replace two-day rolling averages with simple daily returns. Table 4 summarizes the results, showing the number of countries for which interdependence is rejected under each run of the analysis. For each definition of our sample, we carry out the Fisher's test, as well as our test procedure with $\lambda_j = \lambda_j^C$ (where the constant variance ratio is estimated using the 'world stock market index') and with $\lambda_j = \lambda_j^C = 0$.

Our conclusions are quite robust to a change in the currency of denomination of stock prices. This is true not only for countries that maintained a fixed or quasi-fixed exchange rate with respect to the dollar, but also for countries that experienced a sharp devaluation of their currency in our sample period. In the case of Thailand vs. Hong Kong, for instance, $\widehat{\rho}$ and $\widehat{\rho}^C$ are equal to 0.104 and 0.013, respectively, when using returns in local currency, while they are 0.106 and 0.005 when using returns in dollars. When we run our test setting $\lambda_j = \lambda_j^C = 0$, in our benchmark sample we reject interdependence only for Italy; using returns in local currency we also reject interdependence for the UK. When we set $\lambda_j = \lambda_j^C$, our test rejects the null for Italy, the UK, Singapore, France and the Philippines, regardless of the currency in which we calculate returns.

By the same token, our results are robust to changes in the timing of the tranquil and the crisis periods. When we alter the definition of *tranquil period* to include the year 1996, our test rejects the null for Italy, Singapore, France and the Philippines, but not for the UK. As on average correlation remained quite high at the end of 1997 (see Figures 4), we have also estimated a model including December 1997 in the *crisis period*. In this case, results are unaffected relative to our benchmark estimation.

Interestingly, if we replace two-day rolling averages with simple *daily returns*, the number of cases in which the conditional tests reject interdependence increases visibly, both for $\lambda_j = \lambda_j^C = 0$ and for $\lambda_j = \lambda_j^C$.[14] In particular, conditional on $\lambda_j = \lambda_j^C = 0$, we reject interdependence for Italy, France, the UK; using the estimated variance ratio together with the hypothesis $\lambda_j = \lambda_j^C$, we reject also for Singapore, the Philippines, Germany and Russia.[15]

6 Conclusion

This paper presents a unified framework to approach tests of contagion, based on a factor model of returns. We have shown that some leading tests in the literature are conditional on specific yet arbitrary assumptions about a key parameter. This is the ratio between the variance of the country-specific shock and the variance of the global factor weighted by its factor loading. We have shown that the chances of accepting the null of interdependence are very high if one performs correlation tests of contagion conditional on this ratio being constant and equal to zero, $\lambda = 0$.

Our preliminary empirical estimates suggests that, for the case of the Hong Kong stock market crisis in October 1997, the variance of the country-specific

[14]Here we have excluded test results of the United States and Thailand, for which the estimated correlation coefficients during the tranquil and crisis period fall to zero. In this case, tests based on Fisher z-transformation are not appropriate (see Stuart and Ord, 1994).

[15]We also run the same testing procedure as in Forbes and Rigobon (1999a), consisting in a VAR model of returns using domestic and US interest rates as exogenous variables. We have also expanded on their test by including oil prices as exogenous variable. The results from these procedures confirm our conclusions.

component of returns is 2 to 7 times than the variance of global factor. For most country pairs in our sample, however, interdependence can be rejected only for larger values of this ratio. Based on our estimates, we find evidence of contagion from the Hong Kong crisis in the case of Singapore and the Philippines, among the emerging markets, and France, Italy, the UK and (weakly) Germany, among the advanced countries. In contrast, the bias in conditional tests arbitrarily setting $\lambda = 0$ is quite severe. For all the countries in our sample but one, these tests would accept the null of interdependence.

The empirical analysis of this paper has been kept very simple (we used a single factor model of returns), and as close as possible to correlation analysis. It should be clear, however, that the issue of controlling for country-specific shocks in contagion analysis is limited neither to correlation analysis, nor to a single-factor model of returns. In fact, the theoretical part of our paper suggests a direct way of testing contagion vs. interdependence, consisting in a general analysis of structural breaks in multi-factor models of returns contrasting pre-specified tranquil and crisis periods. We leave to future research this development of our analysis.

A Appendix

This appendix derives the expression (2) of the coefficient of interdependence ϕ in the general case. From the data generating process of r_i, the unconditional variance of the idiosyncratic shock ε_i can be written as:

$$Var(\varepsilon_i) = Var(r_i) - \gamma_i^2 \cdot Var(f)$$

By the definition of λ_j and the data generating process of r_j, we can also get:

$$Var(f) = \frac{Var(r_j)}{\gamma_j^2(1+\lambda_j)}$$

Therefore, we find:

$$\frac{Var(\varepsilon_i)}{\gamma_i^2 \cdot Var(f)} = \frac{Var(r_i)}{\gamma_i^2 \cdot Var(f)} - 1 = \frac{\gamma_j^2(1+\lambda_j)Var(r_i)}{\gamma_i^2 Var(r_j)} - 1 \tag{A.1}$$

For convenience, we rewrite the expression of the correlation coefficient induced by the process (1):

$$\rho = \frac{1}{\left[1 + \frac{Var(\varepsilon_i)}{\gamma_i^2 Var(f)}\right]^{1/2} \cdot [1+\lambda_j]^{1/2}} \tag{A.2}$$

Substituting (A.1) into (A.2), we obtain the unconditional correlation coefficient as a function of the rates of return, the factor loadings and λ_j:

$$\rho = \frac{\gamma_i}{\gamma_j}\left[\frac{1}{1+\lambda_j}\left(\frac{Var(r_i)}{Var(r_j)}\right)^{-1/2}\right] \tag{A.3}$$

We now turn to the crisis period. From the data generating process of the rate of return of the stock market in country i, the variance of r_i during the crisis is:

$$Var(r_i \mid C) = \gamma_i^2 \cdot Var(f \mid C) + Var(\varepsilon_i) \tag{A.4}$$

Note that by the data generating process (1) and by the definition of λ_j and λ_j^C, it follows that:

$$\frac{Var(r_j \mid C)}{Var(r_j)} = 1 + \delta = \frac{1+\lambda_j^C}{1+\lambda_j}\frac{Var(f \mid C)}{Var(f)} \tag{A.5}$$

By solving (A.5) for $Var(f \mid C)$ and substituting the resulting expression into (A.4) we get:

$$Var(r_i \mid C) = Var(r_i) + \psi\gamma_i^2 Var(f)$$

where ψ is defined as in follows

$$\psi = \frac{\delta(1+\lambda_j) + (\lambda_j - \lambda_j^C)}{1+\lambda_j^C}$$

Hence, we obtain:

$$\begin{aligned}
\frac{Var(r_i \mid C)}{Var(r_j \mid C)} &= \frac{Var(r_i) + \psi\gamma_i^2 Var(f)}{(1+\delta)Var(r_j)} = \\
&= \frac{Var(r_i)}{(1+\delta)Var(r_j)} + \frac{\psi\gamma_i^2}{(1+\delta)(1+\lambda_j)\gamma_j^2} \qquad \text{(A.6)}
\end{aligned}$$

>From (A.3), the correlation coefficient during the crisis period in the hypothesis that only the variance of f and ε_j change, while the factor loadings remain constant — which is our coefficient of interdependence ϕ — can be written as:

$$\phi(\lambda_j, \lambda_j^C, \delta, \rho) = \frac{\gamma_i}{\gamma_j}\left[\frac{1}{1+\lambda_j^C}\left(\frac{Var(r_i \mid C)}{Var(r_j \mid C)}\right)^{-1/2}\right] \qquad \text{(A.7)}$$

Substituting (A.6) into (A.7), we finally obtain

$$\begin{aligned}
\phi(\lambda_j, \lambda_j^C, \delta, \rho) &= \left[\frac{(1+\lambda_j^C)^2}{(1+\delta)}\frac{\gamma_j^2}{\gamma_i^2}\frac{Var(r_i)}{Var(r_j)} + \frac{\psi(1+\lambda_j^C)^2}{(1+\delta)(1+\lambda_j)}\right]^{-1/2} = \\
&= \left[\frac{(1+\lambda_j^C)^2}{(1+\delta)(1+\lambda_j)^2\rho^2} + \frac{\psi(1+\lambda_j)(1+\lambda_j^C)^2\rho^2}{(1+\delta)(1+\lambda_j)^2\rho^2}\right]^{-1/2} = \\
&= \rho\left\{\frac{(1+\lambda_j^C)^2 \cdot \left[1+\psi(1+\lambda_j)\rho^2\right]}{(1+\delta)(1+\lambda_j)^2}\right\}^{-1/2}
\end{aligned}$$

which can be rearranged to give equation (2).

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Table 1: Average volatility - Crises vs. tranquil periods

crisis	ERM %	Mexican %	Asian %	Rus/Bra %
Korea	-10.0	-32.6	95.9	72.9
Hong Kong	6.2	99.7	158.4	75.7
Indonesia	-72.8	1.3	36.7	7.2
Malaysia	-42.8	16.0	126.9	144.1
Singapore	-39.6	0.0	104.8	77.8
Thailand	-36.0	-21.9	81.5	67.5
Philippines	(-3563)	2.3	81.9	57.9
Argentina	-30.1	-26.6	-34.3	-20.3
Brazil	NA	231.9	183.4	198.4
Mexico	-13.9	121.9	21.9	42.7
Russia	NA	NA	117.9	222.0
United States	-31.2	-34.6	50.8	67.7
Japan	-16.5	-13.8	26.7	9.1
Germany	-25.4	-30.6	49.9	85.8
France	4.4	-3.6	30.3	56.6
United Kingdom	2.1	-20.0	30.7	70.2
Italy	27.0	4.0	39.8	63.5
Canada	-27.9	23.0	51.6	100.8
Euro area	21.7	-0.8	99.5	151.8

Note: the table reports as percentage the square root of the ratio between the average of squared daily returns in each sub-sample to the average in tranquil period less one.

Table 2: Hong Kong crisis - Conditional and Fisher tests

country	$\widehat{\rho}$	$\widehat{\rho}^C$	$\bar{\lambda}$	Fisher	$\frac{1}{\rho^2} - 1$
Indonesia	0.31	0.60	7.1	*	9.7
Korea	0.16	0.07	$+\infty$	-	38.7
Malaysia	0.20	0.43	64.5	-	24.8
Philippines	0.22	0.66	2.6	**	19.8
Singapore	0.36	0.76	1.5	**	6.5
Thailand	0.11	0.01	$+\infty$	-	88.6
Argentina	0.26	0.21	$+\infty$	-	13.5
Brazil	0.20	0.31	30,941.3	-	23.1
Mexico	0.29	0.45	49.3	-	10.8
Russia	0.19	0.53	13.8	*	26.9
USA	0.15	0.26	254.0	-	42.1
Japan	0.28	0.33	7486.5	-	11.4
Germany	0.24	0.63	4.4	**	16.8
France	0.17	0.66	1.2	**	32.3
United Kingdom	0.17	0.63	2.3	-	33.0
Italy	0.00	0.63	0.00	**	732,762
Canada	0.27	0.37	389.8	-	12.8

Note: $\widehat{\rho}$ and $\widehat{\rho}^C$ are estimated correlation coefficients of two-day rolling averages of returns in the tranquil and crisis periods; $\bar{\lambda}$ is the threshold variance ratio as defined in the text (for $\widehat{\delta} = 8.72$). The fourth column reports the results of the Fisher test: * (**) indicates that the hypothesis $\widehat{\rho}^C \leq \widehat{\rho}$ is rejected at the 5 (1) per cent significance level.

Table 3: Estimations of the variance ratio for Hong Kong

	$\lambda = \lambda^C$	λ	λ^C
Cross section:			
G 7	2.8	2.9	3.2
Full sample	2.4	2.6	2.6
World stock market index	3.6	3.0	4.5
Principal components:			
First component	7.1	17.4	2.3
First two components	7.1	17.4	2.1
First three components	6.9	17.3	1.6
First four components	4.9	12.0	1.5

Table 4: Robustness - Test results

	Number of countries for which interdependence is rejected		
Test:	Fisher test	$\lambda = \lambda^C$	$\lambda = \lambda^C = 0$
Sample:			
Benchmark	8	5	1
Local currency	7	5	2
Tranquil: 3.1.96-17.10.97 Crisis: 20.10.97-28.11.97	8	4	1
Tranquil: 3.1.96-17.10.97 Crisis: 20.10.97-31.12.97	8	5	1
Daily returns	8	7	3

The test $\lambda = \lambda^C$ is based on the global factor estimated as the returns on the 'world stock market index'.