

Visualize Inequality:

Inequality of Opportunities using PISA

Overview

Imagine a country where your future did not depend on where you come from, how much your family earns, or whether you are male or female. Imagine if personal circumstances, those over which you have no control or responsibility, were irrelevant to you and your children's opportunities. And imagine now a statistical tool that can help governments make that a reality. That tool is the Human Opportunity Index.

The Human Opportunity Index calculates how personal circumstances (like area of residence, wealth or gender) impact a child's probability of accessing the services that are necessary to succeed in life, like a quality education. With *Visualize Inequality*, the World Bank Group makes available to policy makers, governments, researchers, journalists and the broad audience, the results from several exercises on access to basic opportunities for children.

This documentation file covers the basic aspects of the Human Opportunity Index methodology (section 1), which are not addressed on the individual dashboards. It also presents the data sources (section 2).

1. The Human Opportunity Index

The Human Opportunity Index measures the availability of services that are necessary to progress in life (say, a quality education), discounted by how unfairly the services are distributed among the population. In other words, the Human Opportunity Index is coverage corrected for inequality.

Imagine any country and a basic opportunity based on the Programme for International Student Assessment (PISA): showing the skills in mathematics that enable children to participate effectively and productively in life. If 70 out of 100 show this skills then 70% (that is $\frac{70}{100}$) of children have access to this opportunity –the national coverage rate. Now imagine that 60 of these kids live in urban areas while 40 live in rural areas. However, while the rate of coverage is 90% in urban areas, it is 40% in rural areas. Thus

$$60\% \times 90\% + 40\% \times 40\% = 70\%.$$

We can build a measure of how unequal the access to a high quality education is distributed among groups by computing the absolute value of the difference between the national coverage rate and the group-level coverage rate multiplied by the share of that group in the population, and then adding them up. Therefore, for urban areas the dissimilarity would be

$$60\% \times |70\% - 90\%| = 12\%,$$

and for rural areas the dissimilarity would be

$$40\% \times |70\% - 40\%| = 20\%.$$

Now we proceed to compute our measure of inequality: the Inequality Index, which is the sum of all dissimilarities multiplied by a proportional factor, equal to 0.5 times one divided by the coverage rate. So, we proceed to discount the coverage rate by this measure of inequality:

$$70\% \times \left(100\% - 0.5 \frac{(12\% + 20\%)}{70\%} \right).$$

The result of this operation is 54%, which is the Human Opportunity Index.

More generally, let C be the national coverage rate and C_k the coverage rate for group k defined by a set of circumstances (area of residence, gender, wealth, etc.) $X = (X_1, X_2, \dots, X_n)$, so that $k \in X_1 \times X_2 \times \dots \times X_n$. Therefore, the inequality index D can be defined as:

$$D = \frac{1}{2C} \sum_{k=1}^m \alpha_k |C - C_k|$$

and the Human Opportunity Index (HOI) would be given by

$$HOI = C(1 - D).^1$$

1.1. Inequality Contributors

For each exercise we used the PISA 2009 and 2012 data, which provide comparable data across countries; we also used a set of variables that might be considered exogenous to children (not under their control). However, given that the Human Opportunity Index is sensitive to the circumstances chosen for the analysis, we provide the user with information of the share of the inequality index that is explained by each circumstance. We compute the contribution of each circumstance to inequality by adding and subtracting circumstances from the calculations, so that we are able to determine how important is a given circumstance (for example, wealth) in each calculation. Then we take an average of these numbers to determine the contribution of each circumstance to inequality.

We use Shorrocks (1999) decomposition method to compute the contribution of each circumstance to the inequality index. More formally, consider $D = D(X)$ the inequality index given the vector of circumstances X . If we have two sets of circumstances $A, B \in X$, and $A \cap B = \emptyset$, $D(A, B) \geq D(A)$. Thus, the impact of adding a set of circumstances A is given by:

$$D_A = \sum_{S \subseteq N \setminus \{A\}} \frac{|s|!(n - |s| - 1)!}{n!} [D(S \cup \{A\}) - D(S)]$$

¹ For further methodological details we recommend reading Barros et al. (2009, 2010).

where N is the set of all circumstances, and n is the subset of variables; S is a subset of N (containing s circumstances) that does not contain A . And, thus, we can define the contribution of the set of variables A to the inequality index as $M_A = \frac{D_A}{D(N)}$, where $\sum_{i \in N} M_i = 1$.²

Data Sources PISA:

The following table shows each of the data sources used by country:

Table 1. Data Sources

Country	Survey name	Year
Argentina	Programme for International Student Assessment	2012
Australia	Programme for International Student Assessment	2012
Austria	Programme for International Student Assessment	2012
Azerbaijan	Programme for International Student Assessment	2009
Belgium	Programme for International Student Assessment	2012
Brazil	Programme for International Student Assessment	2012
Bulgaria	Programme for International Student Assessment	2012
Canada	Programme for International Student Assessment	2012
Chile	Programme for International Student Assessment	2012
Chinese Taipei	Programme for International Student Assessment	2012
Colombia	Programme for International Student Assessment	2012
Costa Rica	Programme for International Student Assessment	2012
Croatia	Programme for International Student Assessment	2012
Czech Republic	Programme for International Student Assessment	2012
Denmark	Programme for International Student Assessment	2012
Estonia	Programme for International Student Assessment	2012
Finland	Programme for International Student Assessment	2012
France	Programme for International Student Assessment	2012
Germany	Programme for International Student Assessment	2012
Greece	Programme for International Student Assessment	2012
Hong Kong-China	Programme for International Student Assessment	2012
Hungary	Programme for International Student Assessment	2012
Iceland	Programme for International Student Assessment	2012
Indonesia	Programme for International Student Assessment	2012
Ireland	Programme for International Student Assessment	2012
Israel	Programme for International Student Assessment	2012
Italy	Programme for International Student Assessment	2012
Japan	Programme for International Student Assessment	2012
Jordan	Programme for International Student Assessment	2012

² For further methodological details we recommend reading Hoyos and Narayan (2011).

Kazakhstan	Programme for International Student Assessment	2012
Kyrgyzstan	Programme for International Student Assessment	2009
Korea	Programme for International Student Assessment	2012
Latvia	Programme for International Student Assessment	2012
Liechtenstein	Programme for International Student Assessment	2012
Lithuania	Programme for International Student Assessment	2012
Luxembourg	Programme for International Student Assessment	2012
Macao-China	Programme for International Student Assessment	2012
Malaysia	Programme for International Student Assessment	2012
Mexico	Programme for International Student Assessment	2012
Montenegro	Programme for International Student Assessment	2012
Netherlands	Programme for International Student Assessment	2012
New Zealand	Programme for International Student Assessment	2012
Norway	Programme for International Student Assessment	2012
Panama	Programme for International Student Assessment	2009
Peru	Programme for International Student Assessment	2012
Poland	Programme for International Student Assessment	2012
Portugal	Programme for International Student Assessment	2012
Qatar	Programme for International Student Assessment	2012
Romania	Programme for International Student Assessment	2012
Russian Federation	Programme for International Student Assessment	2012
Serbia	Programme for International Student Assessment	2012
Shanghai-China	Programme for International Student Assessment	2012
Singapore	Programme for International Student Assessment	2012
Slovak Republic	Programme for International Student Assessment	2012
Slovenia	Programme for International Student Assessment	2012
Spain	Programme for International Student Assessment	2012
Sweden	Programme for International Student Assessment	2012
Switzerland	Programme for International Student Assessment	2012
Thailand	Programme for International Student Assessment	2012
Tunisia	Programme for International Student Assessment	2012
Turkey	Programme for International Student Assessment	2012
Trinidad and Tobago	Programme for International Student Assessment	2009
United Arab Emirates	Programme for International Student Assessment	2012
United Kingdom	Programme for International Student Assessment	2012
United States	Programme for International Student Assessment	2012
Uruguay	Programme for International Student Assessment	2012
Vietnam	Programme for International Student Assessment	2012

Source: Poverty Global Practice, Central Unit.

Table 2 shows the opportunities and circumstances used and how are they defined:

Table 2. Opportunities and Circumstances

A. Opportunities

Dimension	Opportunity	Description
Education	Basic skills in math	Percentage of students that demonstrate the basic competencies in math (percentage of students above level 2 in math according to OECD)
	Basic skills in science	Percentage of students that demonstrate the basic competencies in science (percentage of students above level 2 in science according to OECD)
	Basic skills in reading	Percentage of students that demonstrate the basic competencies in reading (percentage of students above level 2 in reading according to OECD)

B. Circumstances

Dimension	Education Opportunities	Infrastructure Opportunities	Health Opportunities
Child Characteristics	Gender	Gender	Gender
	Immigrant	Immigrant	Immigrant
	Attended preeschool	Attended preeschool	Attended preeschool
Household Composition	Presence of parents in the household	Presence of parents in the household	Presence of parents in the household
Location	Area of residence (city size)	Area of residence (city size)	Area of residence (city size)
Parents'/guardians' Characteristics	Education of the highest educated parent/guardian	Education of the highest educated parent/guardian	Education of the highest educated parent/guardian
Socioeconomic status	Quintiles of the economic, social and cultural status index	Quintiles of the economic, social and cultural status index	Quintiles of the economic, social and cultural status index

Notes: An student is defined immigrant if he is first generation immigrant. Presence of parents in the household has three categories: presence of both parents, presence of either mother or father, or absence of father and mother. Area of residence has the following categories: village (less than 3000 hab.); small town (3000 to 15000 hab.); town (15000 to 100000 hab.); city (100000 to 1000000 hab.), and large city (more than 1000000 hab.). Education of the highest educated parent/guardian has the following categories: no education; primary school; secondary school, and tertiary education.

Source: Poverty Global Practice, Central Unit.

References

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